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Large rescaling of the scalar condensate, towards a Higgs-gravity connection ? [1]

J.J. van der Bij
Fakultät für Physik,
Albert–Ludwigs–Universität Freiburg,
Hermann–Herder–Strasse 3,
79104 Freiburg, Germany.

Abstract

In the Standard Model the Fermi constant is associated with the vacuum expectation value of the Higgs field $\langle\Phi\rangle$, 'the condensate', usually believed to be a nearly cut-off independent quantity. General arguments related to the 'triviality' of $\lambda\Phi^4$ theory in 4 space-time dimensions suggest, however, a dramatic renormalization effect in the continuum theory. This effect is visible on the relatively large lattices (such as 32^4) available today. The result is suggestive of a certain 'Higgs-gravity connection', as discussed some years ago. The space-time structure is determined by symmetry breaking and the Planck scale is essentially a rescaling of the Fermi scale. The resulting picture may lead to quite substantial changes in the usual phenomenology associated with the Higgs particle.

The aim of this paper is to analyze the scale dependence of the 'Higgs condensate' $\langle\Phi\rangle$, an important subject for any theory aiming to insert the electroweak theory in more ambitious frameworks. At the same time, it turns out that the solution of this genuine quantum-field-theoretical problem can provide precious insights on possible connections between the Higgs sector and spontaneously broken theories of gravity. We shall restrict to the case of a pure Φ^4 theory and comment in the end about the stability of the results when other interactions (such as gauge and yukawa) are present.

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Let us define the theory in the presence of a lattice spacing $a \sim 1/\Lambda$ and assume that the ultraviolet cutoff Λ is much larger than the Higgs mass M_h. Our basic problem is to relate the bare condensate

$$v_B(\Lambda) \equiv \langle \Phi_{\rm latt} \rangle \tag{1}$$

to the low-energy physical value v_R (that in the Standard Model is associated with the Fermi scale $v_R \sim 246$ GeV). In the usual approach, one writes

$$v_B^2 = Z v_R^2 \tag{2}$$

where Z is called the 'wave-function renormalization'.

As pointed out in [1], in the presence of spontaneous symmetry breaking, where there is no smooth limit $p \to 0$, there are two basically different definitions of Z:

a) $Z \equiv Z_{\rm prop}$ where $Z_{\rm prop}$ is defined from the propagator of the shifted fluctuation field $h(x) = \Phi(x) - \langle \Phi \rangle$, namely

$$G(p^2) \sim \frac{Z_{\rm prop}}{p^2 + M_h^2} \tag{3}$$

b) $Z \equiv Z_\varphi$ where Z_φ is the rescaling needed in the effective potential $V_{\rm eff}(\varphi)$ to match the quadratic shape at its absolute minima with the Higgs mass, namely

$$\frac{d^2 V_{\rm eff}}{d\varphi_R^2}|_{\varphi_R = v_R} = M_h^2 \tag{4}$$

Now, by assuming 'triviality' as an exact property of Φ^4 theories in four space-time dimensions [2], it is well known that the continuum limit leads to $Z_{\rm prop} \to 1$. However, is this result relevant to evaluate the scale dependence of the Higgs *condensate* ? When dealing with the space-time constant part of the scalar field, it is $Z \equiv Z_\varphi$, as defined from Eq.(4), that represents the relevant definition to be used in Eq.(2).

Now, as pointed out in [1], by restricting to those approximations to the effective potential (say $V_{\rm eff} \equiv V_{\rm triv}$) that are consistent with 'triviality', since the field $h(x)$ is governed by a quadratic hamiltonian and $Z_{\rm prop} = 1$, one finds a non-trivial Z_φ. Indeed, $V_{\rm triv}$, given by the sum of a classical potential and the zero-point energy of the shifted fluctuation field, is an extremely *flat* function of φ_B implying a divergent $Z_\varphi \sim \ln \frac{\Lambda}{M_h}$ in the limit $\Lambda \to \infty$. Thus, when properly understood, 'triviality' requires at the same time $Z_{\rm prop} \to 1$ and $Z_\varphi \to \infty$, implying that, in a true continuum theory, v_B/v_R can become arbitrarily large.

The existence of a non-trivial Z_φ for the Higgs condensate, quite distinct from the $Z_{\rm prop}$, associated with the finite-momentum fluctuation field, is a definite prediction that

can be tested with a precise lattice computation. To this end a lattice simulation of the Ising limit of Φ^4 theory was used [3] to measure i) the zero-momentum susceptibility:

$$\chi^{-1} = \frac{d^2 V_{\rm eff}}{d\varphi_B^2}|_{\varphi_B = v_B} \equiv \frac{M_h^2}{Z_\varphi} \tag{5}$$

and ii) the propagator of the shifted field (at Euclidean momenta $p \neq 0$). The latter data can be fitted to the form in Eq.(3) to obtain M_h and $Z_{\rm prop}$.

The lattice data show that $Z_{\rm prop}$ is slightly less than one, and tends to unity as the continuum limit is approached, consistently with the non-interacting nature of the field $h(x)$. However, the Z_φ extracted from the susceptibility in the broken phase is clearly different. It shows a rapid increase above unity and the trend is consistent with it diverging in the continuum limit. Absolutely no sign of such a discrepancy is found in the symmetric phase, as expected. The first indications obtained in [3] are now confirmed by a more complete analysis [4] performed with more statistics on the largest lattices employed so far, such as 32^4. The results accord well with the "two-Z" picture where one predicts a $Z_{\rm prop}$ slowly approaching one and a zero-momentum rescaling $Z_\varphi = M_h^2 \chi$ becoming higher and higher the closer we get to the continuum limit.

The above result, giving a large difference in the rescalings of the vacuum and the fluctuation fields, has been established for the model with a single scalar field. The results have been described on the lattice. In order to make contact with experiment, one must extend the results to the O(4) sigma model, which describes the Higgs sector of the standard model. This can be done straightforwardly when one has a description of the effect directly in the continuum. To describe the different rescalings in the continuum one should consider the effective Lagrangian, which is in general a combination of many operators. We are looking for an operator that would give rise to a rescaling of the Higgs propagator in the broken phase but not in the unbroken phase. At the same time it should leave the constant fields untouched and be invariant under the symmetry of the theory. This leads us to consider the following operator: $(\partial_\mu |\Phi|^2)(\partial^\mu |\Phi|^2)$. Because of the derivatives this operator leaves the constant fields untouched, but leads to a wave function renormalization of the Higgs field after spontaneous symmetry breaking. To make a connection with the standard model one now simply takes Φ to be the four component Higgs field.

If one adds this term to the standard model as an effective interaction, with a very large coefficient, the result after symmetry breaking is very simple. In the unitary gauge one finds simply the standard model, but with a Higgs coupling to matter reduced by the wave function renormalization. Ultimately, when one takes the wave function renormalization

to ∞, the Higgs particle does not couple to matter anymore. One therefore ends up with the standard model without Higgs-particle. Given the precision of present data in the weak interactions one has to ask oneself whether this picture is in agreement with experiment. At the fundamental level there is clearly a problem reconciling perturbative calculations of electroweak parameters like $\delta\rho$ with the demands of triviality in the scalar sector. To do perturbative calculations one simply needs the Higgs matter and self-couplings to get finite results. However presently we are only sensitive to one-loop corrections, where these couplings do not play a role. At the one loop level the Higgs particle plays essentially a role as a cut-off to the momentum integrals. Corrections to electroweak quantities behave like $log(m_H/m_W) + c$. The logarithmic corrections are universal, in the sense that essentially any form of dynamical symmetry breaking would give rise to logarithmic divergences with the same coefficients as the $log(m_H)$ terms. It is only the constants, that are unique for the normal perturbative interpretation of the standard model. While present data indicate a low value of the Higgs mass within the standard perturbative calculations, they are not precise enough to determine the constant terms uniquely. Therefore a model without a Higgs, but with a cut-off around the weak scale, implying new interactions, cannot be excluded at present.

The origin of the triviality, as found above from the lattice data, lies in the appearance of a very large, essentially infinite, ratio of wave function renormalizations. This naturally brings up the question whether there might be a connection with other large ratios of parameters that appear in fundamental physics. Here we suggest a connection with the ratio of Planck mass and Fermi scale. There are a number of reasons that suggest that a connection between gravity and the Higgs sector is possible. There is for instance the question of the cosmological constant, which is generated by the Higgs potential. Another argument is the fact, that both gravity and the Higgs particle have a universal form of coupling to matter. Gravity couples universally to the energy-momentum tensor, the Higgs particle to mass, which corresponds to the trace of the energy-momentum tensor. However the way the Planck scale appears is quite different from the way the weak scale appears. This discrepancy can be resolved by assuming that also the Planck scale arises after spontaneous symmetry breaking.

We start therefore with the Lagrangian:

$$\mathcal{L} = \sqrt{g}(\xi\Phi^+\Phi R - \frac{1}{2}g^{\mu\nu}(D_\mu\Phi)^+(D_\nu\Phi) - V(\Phi^+\Phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}) \tag{5}$$

This is the spontaneous symmetry breaking theory of gravity [5], with the standard

model Higgs v.e.v. as the origin of the Planck mass. The parameter ξ is given by $\xi = 1/16\pi G_N v^2$. This parameter is of $O(10^{34})$ and gives rise to a large mixing between the Higgs boson and the graviton. This model was recently discussed in [6]. The physical content of the model becomes clear after the Weyl rescaling $g_{\mu\nu} \to \frac{\kappa^2}{\xi|\Phi|^2} g_{\mu\nu}$, giving the Lagrangian :

$$\mathcal{L} = \sqrt{g}(\kappa^2 R - \frac{3}{2}\frac{\xi v^2}{|\Phi|^4}(\partial_\mu|\Phi|^2)(\partial^\mu|\Phi|^2) - \frac{1}{2}\frac{v^2}{|\Phi|^2}(D_\mu\Phi^+)(D^\mu\Phi) - \frac{v^4}{|\Phi|^4}V(|\Phi|^2)) \qquad (6)$$

As ξ is very large indeed, this final Lagrangian is of the right form to be consistent with the triviality of the Higgs sector. After making the wavefunction renormalization by the factor $(1 + 12\xi)^{1/2}$ the coupling of the Higgs particle to ordinary matter becomes of gravitational strength. Also the mass of the Higgs particle becomes of the order v^2/m_P. This would give rise to a Yukawa potential of gravitational strength with a range that could be in the millimeter range. Such a force can be looked for at the next generation of fifth force experiments.

If indeed such a close connection between gravity and the Higgs sector is present in nature, quite dramatic effects should be expected at the electroweak scale, as quantum gravity would already appear at this scale. What these effects are precisely, cannot be predicted confidently at the present time. Typical suggestions are higher dimensions, string Regge trajectories, etcetera. In any case it is very suggestive that the picture of triviality in the Higgs sector, can be so naturally described in the spontaneous symmetry breaking theory of gravity.

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